## LETTERS TO THE EDITORS

## Statistics of slide blocks

## Dr. J. F. Savage writes:

In the article by Naylor (1982), the statistics of the orientation of the long axes of slide blocks are not adequately treated so that the deductions about them are not correct.

The rose diagram in fig. 7(a) of Naylor (1982) shows two clearly defined maxima at an angle of $60^{\circ}$ to each other which immediately suggests to me a multimodal character for their orientations. The maxima are relatively strong (c. $\mathbf{1 7 \%}$ ) whereas the resultant vector magnitude is only just significant at the $95 \%$ level. Implicit in the Raleigh Test of uniformity of the test statistic $R$, which is used by Naylor (1982), the mean resultant vector magnitude, is a unimodal model for the distribution being tested against a uniform one. Where, for any reason, the unimodal model is suspect the test cannot be considered appropriate for assessing deviations from uniformity. The difficulty can be largely alleviated by applying a non-parametric test, a number of which are capable of assessing multimodal data. Where necessary it is then possible to carry out transformations to make multimodal data amenable to additional analysis.

The most appropriate and probably most powerful test has been dubbed the Kuiper Test by Mardia (1972) who gives a description of the theory and its practice on which this account is based. This is a Kolmogorov-type test which Kuiper (1960) and Stephens (1965, 1970) have developed so that it is relatively easy to apply. The example has been reworked using the original data supplied by Dr. Naylor, the work sheet being given in Figs. 1 and 2 .

The data are orientations without a sense of direction (the axial type) and the measurements, as tabulated, are effectively restricted to a $180^{\circ}$ range (modulo $\pi$ ) (Mardia 1972, p. 7). To obtain truly circular parameters it is first necessary to double the angles (note that there are a number of directions with many readings, which seems to be related to the reading technique and not the result of grouping). The graph of


Fig. 1. Application of Kuipers test to Slide Block data, (Modulo $\pi$ ) (Naylor 1982). Calculation of $V^{*}$ statistic (Mardia 1972). Mean Vector Magnitude $\bar{R}=0.335$ (significant at 0.05 level). Mean Vector Orientation 161.8.

$$
\begin{aligned}
V_{n} & =\frac{D_{\max }^{+}-D_{\min }^{-}+1}{n}=\frac{7.1-(-5.48)+1}{39} \\
& =0.348 . \\
V^{*} & =\left(V n+0.155+0.24 \frac{1}{\sqrt{n}}\right) V_{n} \\
& =\left(\sqrt{ } 39+0.155+\frac{0.24}{\sqrt{3} 39}\right) \times 0.348 \\
& =2.24 \text { (very highly significant) } .
\end{aligned}
$$

cumulative frequencies can now be made as shown; it does not have to be recalculated to probability ratios as in most statistical accounts. The diagonal of this graph represents the uniform model of equal accretion per unit of orientation. The maximum deviations in a negative and positive direction. $D_{n}{ }^{+}$and $D_{n}{ }^{-}$, can be either read off the graph or, for unnecessary accuracy calculated from the slope of the line and a given cumulative total (Mardia 1972, table 7.2, p. 179).

Table 1. Table of significance levels for the statistic $V^{*}$ (after Stephens 1970)

| $\boldsymbol{\alpha}$ | 0.10 | 0.05 | 0.025 | 0.01 |
| :--- | :--- | :--- | :--- | :--- |
| $V^{*}$ | 1.62 | 1.747 | 1.862 | 2.001 |

The data, treated as on a $180^{\circ}$ range $(\operatorname{Mod} \pi)$, yield statistics $V^{*}=$ 2.24 which can be seen from Table 1 to be very highly significant ( $P\left(V_{n}^{\prime}=2.338\right) \ll 0.005$ ). This contrasts with the significance of the $\bar{R}$ statistic for these data: the value of 0.335 the significance of which lies somewhere between $P_{0.025}$ and $P_{0.01}$ (Mardia 1972, App. 2.5). This divergence of indications is what would normally be considered suspect and require specific explanation.

In order to take into account a possible cyclicity with a $60^{\circ}$ period it is necessary to further transform the data by multiplying them on the $360^{\circ}$ range by three (modulo $\pi / 3$ ). The direction of the mean vector has to be transformed back to the original scale of the measurements and this results in three multiples, $60^{\circ}$ apart (Fig. 2). The resulting frequency distribution yields a much higher value, 0.477 , for the magnitude of the mean resultant vector, $\bar{R}$. This has a significance much higher than the tabulated value for $P=0.001$ in Mardia's extended tables. The value of the Kuiper statistic $V^{*}(=2.72)$ is only a little higher than that for the untransformed data which demonstrates how sensitive this test can be for multimodal distributions even when they have not been transformed.


Fig. 2. Application of Kuipers test to Slide Block Data (Modulo $\pi / 3$ ) (Naylor 1982). Mean Vector Orientations 004.5; 064.5; 124.5. Mean Vector Magnitude $\tilde{R}=0.477$ (very highly significant).

$$
\begin{aligned}
V_{n} & =\frac{D_{\max }^{+}-\left(D_{\min }^{-}\right)+1}{n}=\frac{13.0-(-2.5)+1}{39}=0.423 . \\
V^{*} & =\left(V_{n}+0.155+0.24 \frac{1}{\sqrt{n}}\right) \\
& =\left(\sqrt{ } 39+0.155+\frac{0.24}{\sqrt{39}}\right) \\
& =2.72 \text { (very highly significant) } .
\end{aligned}
$$

Table 2. Critical values of $\bar{R}$ (Raleigh test) for $n=39$ (after Stephens 1969)

| $\alpha$ | 0.05 | 0.01 |
| :--- | :--- | :--- |
| $\bar{R}$ | 0.275 | 0.341 |

We can now conclude that the distribution of the azimuths of the long axes of the slide blocks discussed by Naylor (1982) are almost certainly non-random (-uniform) and that an appropriate model can be formed by assuming a cyclicity of $60^{\circ}$ between clusters of measurements. The clusters are centred around the azimuths 004,064 and $124^{\circ}$ on the $180^{\circ}$ range. What this model means in geological terms is not at all clear but any comment on these slide blocks should take this pattern of preferred orientation into account.
Instituut voor aardwetenschappen
J. F. SAVaGE

Rijksuniversiteit Utrecht
3508 TA Utrecht
The Netherlands

## Dr. M. A. Naylor replies:

I endorse Dr. Savage's demonstration of the usefulness of the simple Kuiper test in seeking preferred orientations in two-dimensional orientation data.
However, it should be noted that one of the three modes (at $064^{\circ}$ ) implied by the statistical treatment of Dr. Savage is not borne out by the original data (fig. 7(a) of Naylor 1982). The remaining two modes have a straightforward explanation. Inspection of the map of the Casanova Complex (fig. 2 of Naylor 1982) reveals that the slide blocks commonly occur in groups with similar long axis orientations. This does not contradict the hypothesis, based on several other lines of evidence, that the blocks were emplaced by gravity sliding. Indeed, the
grouping of blocks suggests that the first block which came to rest determined (presumably by its relief) the final orientation of successive elongate blocks which came to rest against it. The clustering of blocks may be due to their accumulation in localized depressions in the sedimentary basin (ponding) or to localized, fault-bounded source areas within an otherwise more extensive, unbroken and uniformly dipping distal continental margin.

To alleviate the interdependence of block orientations within the geographic groups, one could attribute the same weight to the mean long axis direction of a group and to each single isolated block. The case for random (uniform) orientation of slide block long axes then becomes much stronger, using either the Rayleigh or Kuiper test.

Exploration Department
Michael A. Naylor
Petroleum Development Oman
P.O. Box 81

Muscat
Sultanate of Oman

## CONSOLIDATED REFERENCES

Kuiper, N. H. 1960. Tests concerning random points on a circle. Ned. Akad. Wet. Proc. A63, 38-47.
Mardia, K. V. 1972. Statistics of Directional Data. Academic Press, New York.
Naylor, M. A. 1982. The Casanova Complex of the Northern Apennines: a mélange formed on a distal passive continental margin. $J$. Struct. Geol. 4, 1-18.
Stephens, M. A. 1965. The goodness-of-fit statistic $V_{n}$ : distribution and significance points. Biometrika 52, 309-321.
Stephens, M. A. 1970. Use of the Kolmogorov-Smirnov, Cramer-Von Mises and related statistics without extensive tables. J. R. Statist. Soc. B32. 115-122.

